

Üçgen Bir Kavite İçerisindeki Bakır-Su Nanoakışkanların Doğal Konveksiyon Üzerinde Viskozite Modellerinin Etkisi

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Özet

Bu çalışmada, üçgen bir kavite içindeki bakır-su bazlı nanoakışkanların laminer doğal konveksiyon akışı sonlu hacimler metoduyla nümerik olarak incelenmiştir. Kavitenin yan duvarları farklı şekilde ısıtılmış olup, alt duvarı dalgali ve yalıtılmıştır. Yönetici parametre olarak Grashof sayısı $10^4 - 10^6$ aralığında, katı partiküllerin hacim fraksiyonu 0, 0.02, 0.04 aralığında ele alınmıştır. Nanoakışkanların viskozitesinin hesaplanmasında Batchlor, Brinkman, Pak ve Cho, Einstein and Maiga vd. modelleri kullanılmıştır. Sonuçlar göstermektedir ki, ısı transfer miktarı, katı hacim fraksiyonu ve Grashof sayısının artmasıyla neredeyse lineer olarak artmaktadır. En yüksek ısı transfer oranı Batchelor modeli kullanıldığında ortaya çıkmaktadır.

Anahtar Kelimeler: Doğal konveksiyon, Nanoakışkan, Üçgen Kavite, Viskozite model.

Effects of Viscosity Models on Natural Convection of Cu-water Nanofluids in a Triangular Cavity

Abstract

In this study, laminar natural convection heat transfer and fluid flow in a triangular cavity filled with Cu-water nanofluids is studied numerically by using finite volume method. Bottom wall of the cavity is taken as adiabatic while inclined walls have different temperature which are isothermally heated. As governing parameters, Grashof number is taken in the range of $10^4 - 10^6$, the solid particle volume fraction values are taken as 0, 0.02 and 0.04. In the calculation, different viscosity models such as Batchelor, Brinkman, Pak and Cho, Einstein and Maiga et al. are tested on natural convection heat transfer and fluid flow. It is observed that heat transfer increases almost linearly with increasing of solid volume fraction and Grashof number. Also, another finding is that the highest heat transfer is observed when Batchelor model is used.

Keywords: Natural convection, Nanofluid, Triangular cavity, Viscosity model.

1. Introduction

Curvilinear shaped enclosures can be seen in many engineering applications such as cooling of electronic equipments, building design and heat exchanger design. Curvilinear boundary brings some difficulties on numerical solution due to problems on boundaries and chosen grids. As an example of natural convection in triangular enclosures, Oztop et al. [1] solved a problem on natural convection in right angle triangular enclosure filled with saturated cold water which has a density maximum around 4 °C. They showed that heat transfer decreases

with the effects of density inversion and it also decreases with increasing of aspect ratio. Varol et al. [2] worked on entropy generation and natural convection in isosceles triangular enclosures with partially heated from below and symmetrically cooled from sloping walls. They used finite difference method to solve governing equations and found that both entropy production due to heat transfer and fluid friction irreversibility are affected by higher inclination angle of triangle and length of heater. Other applications on natural convection in differentially heated triangular cavities for

building design can be seen in literature as Haese and Teubner [3], Asan and Namlı [4].

Nanotechnology is increased rapidly in recent years in many technologies. Nanofluids can be used to control heat transfer and fluid flow in forced and natural convection heat transfer area. Cu–water nanofluid filled triangular enclosure with a rotating cylinder is investigated numerically under magnetic field by Selimefendigil and Oztop [5]. In their work, a partial heater is added on the left vertical wall of the cavity and the right inclined wall is kept at constant temperature. They found that 50.4% and 37.4% of heat transfer enhancements are obtained for $v = 20$ and compared to motionless cylinder $v = 0$. A further observing indicated that heat transfer and entropy generation increase with the solid volume fraction of nanoparticle increases.

Rahman et al. [6] performed a work on unsteady natural convection heat transfer in an isosceles triangular enclosure filled with Al_2O_3 nanoparticle by using finite element method. In the study, the bottom of the isosceles triangular cavity is heated non-uniformly and temperature of the inclined wall is lower than that of the bottom wall. They found that addition of the nanoparticle into base fluid (water) affects both heat transfer and fluid flow. Heat transfer increases with addition of nanoparticle and increasing of Rayleigh number.

Variation of thermal properties such as thermal conductivity ratio, viscosity or specific heat plays important role on nanofluid filled systems. In this context, a wide review has been performed to show the effects of viscosity of nanofluids by Meyer et al. [7]. They showed that the important parameters influencing viscosity of nanofluids are temperature, nanoparticle volume fraction, size, shape, pH, and shearing rate. Regarding the composite of nanofluids, which can consist of different fluid bases and different nanoparticles, different accurate correlations for different nanofluids need to be developed. Abdellahoum et al. [8] performed a work on viscosity variation formulations for turbulent flow of nanofluid flow over a heated cavity in a duct. They tested five different viscosity models such as Pak and Cho, Maiga et al, Einstein, Brinkman and Batchlor. They observed that the Einstein, Brinkman and Batchlor viscosity

models show similar results and lesser friction and heat transfer. Pak and Cho viscosity model gives maximum friction and heat transfer. Mansour and Ahmed [9], Rahman et al. [10], Billah et al. [11] and Mahmoudi et al. [12] studied on effects of different parameters on natural convection in triangular enclosures filled with nanofluids.

The main objective of this work is to present the effects of viscosity variation in an isosceles triangular enclosures heated and cooled with isothermally from its inclined wall. Investigation of viscosity variation on natural convection in triangular enclosures with different isothermal inclined walls is the main novelty for this work.

2. Mathematical Formulation

The physical model is defined in Fig. 1. Inclined walls have isothermal and bottom side is adiabatic. Temperature of left inclined side is higher than that of right one. The gravity acts in $-y$ direction. Height and length of the cavity are given by H and L , respectively.

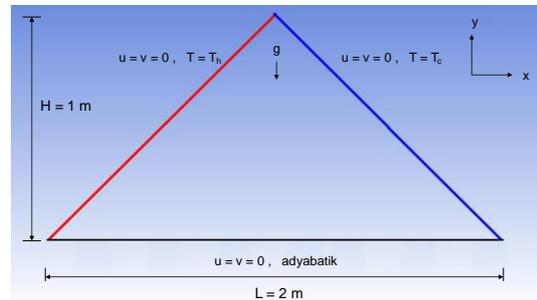


Fig 1. Geometry and boundary conditions

Water based Cu was chosen as nanofluid. Its properties are given in Table 1.

Table 1. Thermophysical properties

Properties	ρ (kg/m^3)	C_p (J/kgK)	k (W/mK)	$\alpha \times 10^7$ (m^2/s)	$\beta_T \times 10^6$ ($1/K$)
Water	997.1	4179	0.613	1.47	210
Cu	8933	385	400	1163	16.7

Governing equations of natural convection is given for two-dimensional, steady and laminar flow with Boussinesq approximation as follow:

continuity

$$\frac{\partial \mathbf{u}^*}{\partial x^*} + \frac{\partial \mathbf{v}^*}{\partial y^*} = 0 \quad (1)$$

x-momentum equation

$$\left(\mathbf{u}^* \frac{\partial \mathbf{u}^*}{\partial x^*} + \mathbf{v}^* \frac{\partial \mathbf{u}^*}{\partial y^*} \right) = -\frac{1}{\rho_{nf,0}} \frac{\partial p^*}{\partial x^*} + \frac{\mu_{eff}}{\rho_{nf,0}} \left(\frac{\partial^2 \mathbf{u}^*}{\partial x^{*2}} + \frac{\partial^2 \mathbf{u}^*}{\partial y^{*2}} \right) \quad (2)$$

y-momentum equation

$$\left(\mathbf{u}^* \frac{\partial \mathbf{v}^*}{\partial x^*} + \mathbf{v}^* \frac{\partial \mathbf{v}^*}{\partial y^*} \right) = -\frac{1}{\rho_{nf,0}} \frac{\partial p^*}{\partial y^*} + \frac{\mu_{eff}}{\rho_{nf,0}} \left(\frac{\partial^2 \mathbf{v}^*}{\partial x^{*2}} + \frac{\partial^2 \mathbf{v}^*}{\partial y^{*2}} \right) + \frac{1}{\rho_{nf,0}} (\rho\beta)_{nf} g (T - T_C) \quad (3)$$

energy equation

$$\left(\mathbf{u}^* \frac{\partial T}{\partial x^*} + \mathbf{v}^* \frac{\partial T}{\partial y^*} \right) = \alpha_{nf} \left(\frac{\partial^2 T}{\partial x^{*2}} + \frac{\partial^2 T}{\partial y^{*2}} \right) \quad (4)$$

Dimensionless parameters are listed as

$$x = \frac{x^*}{H}, y = \frac{y^*}{H}, u = \frac{u^*}{\alpha_f / H}, \tau = \frac{\alpha_f t}{H^2}, p = \frac{H^2}{\rho_{f0} \alpha_f^2}, \quad (5)$$

$$v = \frac{v^*}{\alpha_f / H}, \theta = \frac{T - T_C}{T_H - T_C}$$

In above equations, u^* and v^* stand for velocities in x^* and y^* directions and p^* shows dimensional pressure, T^* dimensional temperature, ρ_{f0} is density at T_C and α_f is thermal diffusivity of the fluid. Prandtl and Grashof numbers are given as

$$Pr = \frac{\mu_f}{\rho_{f,0} \alpha_f}, Gr = \frac{\rho_{f,0}^2 g \beta_f H^3 (T_H - T_C)}{\mu_f^2} \quad (6)$$

In above equations, μ is dynamic viscosity, β is thermal expansion coefficient and α is given as thermal diffusivity. Viscosity of the nanofluid can be explained by using two-phase mixing. In this study, spherical solid particles are injected in base fluid. Thus, different viscosity models can be explained as

Batchlor [13],

$$\mu_{eff} = \mu_f (1 + 2.5\phi + 6.5\phi^2) \quad (7)$$

Brinkman [14],

$$\mu_{eff} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (8)$$

Einstein [15],

$$\mu_{eff} = \mu_f (1 + 2.5\phi) \quad (9)$$

Maiga et al. [16]

$$\mu_{eff} = \mu_f (1 + 7.3\phi + 123\phi^2) \quad (10)$$

Pak and Cho [17]

$$\mu_{eff} = \mu_f (1 + 39.11\phi + 533.9\phi^2) \quad (11)$$

Density, thermal capacity, thermal expansion coefficient and thermal diffusivity of the nanofluids are given as

$$\rho_{nf,o} = (1 - \phi)\rho_{f,o} + \phi\rho_{s,o} \quad (12)$$

$$(\rho c_p)_{nf} = (1 - \phi)\rho_f c_{pf} + \phi\rho_s c_{ps} \quad (13)$$

$$(\rho\beta)_{nf} = (1 - \phi)\rho_f \beta_f + \phi\rho_s \beta_s \quad (14)$$

$$\alpha_{nf} = \frac{k_{eff}}{(\rho c_p)_{nf,o}} \quad (15)$$

In above equations, ϕ is the volume fraction of nanoparticle and eff, f ve s indicate the particles for nanofluid, liquid and solid, respectively. Also, different models are tested for thermal conductivity as,

Yu and Choi model [18]

$$\frac{k_{eff}}{k_f} = \frac{k_s + 2k_f + 2(k_s - k_f)(1 + \eta)^3 \phi}{k_s + 2k_f - (k_s - k_f)(1 + \eta)^3 \phi} \quad (16)$$

where η is the ratio of the nanolayer thickness to the original particle radius, is taken equal to 0.1.

The local Nusselt number along the hot wall of the enclosure can be expressed as:

$$Nu = -\frac{k_{eff}}{k_f} \frac{\partial \theta}{\partial n} \quad (17)$$

where n is indicate the direction perpendicular to the surface.

The average Nusselt number on the hot wall can then takes the following form:

$$Nu_a = -\frac{k_{eff}}{k_f} \left(\int_0^{L/2} \frac{\partial \theta}{\partial x} dx + \int_0^{\sqrt{2}L/2} \frac{\partial \theta}{\partial y} dy \right). \quad (18)$$

3. Numerical Method and Validation

In the study ANSYS Fluent 14.0 is used to solve the governing equations. Momentum and energy equations are discretized by second order upwind scheme the pressure-velocity equation is coupled by SIMPLE algorithm.

Table 2. Comparison of present results of mean Nusselt number with literature for $Gr=10^5$

	Particle/ ϕ	0	0.05	0.08	0.1
Present	Cu	10.0	11.6	12.6	13.1
Rahman et al. [10]	Cu	10.3	11.7	12.3	12.8
Present	TiO ₂	10.0	11.1	11.7	12.1
Rahman et al. [10]	TiO ₂	10.3	11.4	11.9	12.3

In the present work, Grashof number values are taken in the range of 10^4 - 10^6 , volume fraction of nanofluid is taken as 0, 0.02 ve 0.04. Five different viscosity models are accepted as Einstein [15], Brinkman[14], Batchlor [13], Maiga et al. [16] and Pak and Cho[17]. Yu and Choi [18] model is chosen to calculate heat conduction coefficient for Cu-water nanofluid.

4. Results and Discussion

A computational study has been performed to work on effects of different viscosity models in a triangular enclosure for different Grashof number and nanoparticle volume ratio.

Fig. 2 illustrates the effects of governing parameters such as nanoparticle volume fraction and Grashof number on fluid flow. The main circulation cell rotates in clockwise direction due to moving nanofluid from the heated side and single cell is formed for lower Grashof numbers. On the contrary double circulation cells are formed for $Gr = 10^6$. As seen from the figure, nanoparticle volume fraction makes less effect on streamlines. However, the flow strength increases with increasing of nanoparticle volume fraction.

Isotherms are illustrated in Fig. 3 for different Grashof numbers and nanoparticle volume fraction. As shown in Fig. 1 that the cavity is heated from left inclined wall isothermally. Thermal boundary layer becomes thinner with increasing of Gr number. Also, most of the volume is heated with increasing of Grashof number. Isotherms become parallel to bottom wall at the highest value of Gr number. It means that convection heat transfer become dominant onto conduction. Heated volume is increased with increasing of nanoparticle volume fraction. It is clearly seen from the Fig. 4 that heat transfer increases with nanoparticle volume fraction almost linearly and the highest mean

Nusselt number is formed for the highest value of Grashof number as expected due to increasing of kinetic energy inside the cavity. As indicated in Fig. 5 that application of viscosity model plays important role on heat transfer. The figure presents effects of viscosity models for $Gr = 10^4$. Five different viscosity model was used in this work as Batchlor, Brinkman, Einstein, Maiga and Pak and Cho. For all models, linear increasing on heat transfer is seen with nanoparticle volume fraction but the trend of this model becomes highest for Pak and Cho. Values for Maiga et al. model sits between Pak and Cho and other models. Heat transfer results give almost same results for remaining three models. In similar manner, mean Nusselt numbers for $Gr = 10^5$ are presented in Fig. 6. The similar trend is observed for this Gr numbers but higher Nusselt number values are formed. Again, linear distribution is formed for Fig. 7 at the highest value of Grashof number. Fig. 8 (a) to (c) illustrates the local Nusselt numbers for different values of nanoparticle volume fraction with Pak and Cho viscosity models at different Grashof numbers. The maximum value of local Nusselt number increases with increasing of nanoparticle volume fraction. Higher values are formed at the middle of the heated wall.

5. Conclusions

In this study, a computational work has been done on natural convection in a triangular enclosure with different viscosity models as Batchlor, Brinkman, Einstein, Maiga et al. and Pak and Cho. The main findings from the studied work can be drawn as

- The maximum Nusselt number was obtained in case of using of Pak and Cho viscosity model.
- Addition of nanoparticle enhances the heat transfer.
- Heat transfer enhances with increasing of nanoparticle volume fraction.
- Heat transfer increases with increasing of Grashof number due to increasing of kinetic energy. Thus, conduction heat transfer becomes dominant to convection for lower values of Grashof numbers.

Effects of Viscosity Models on Natural Convection of Cu-water Nanofluids in a Triangular Cavity

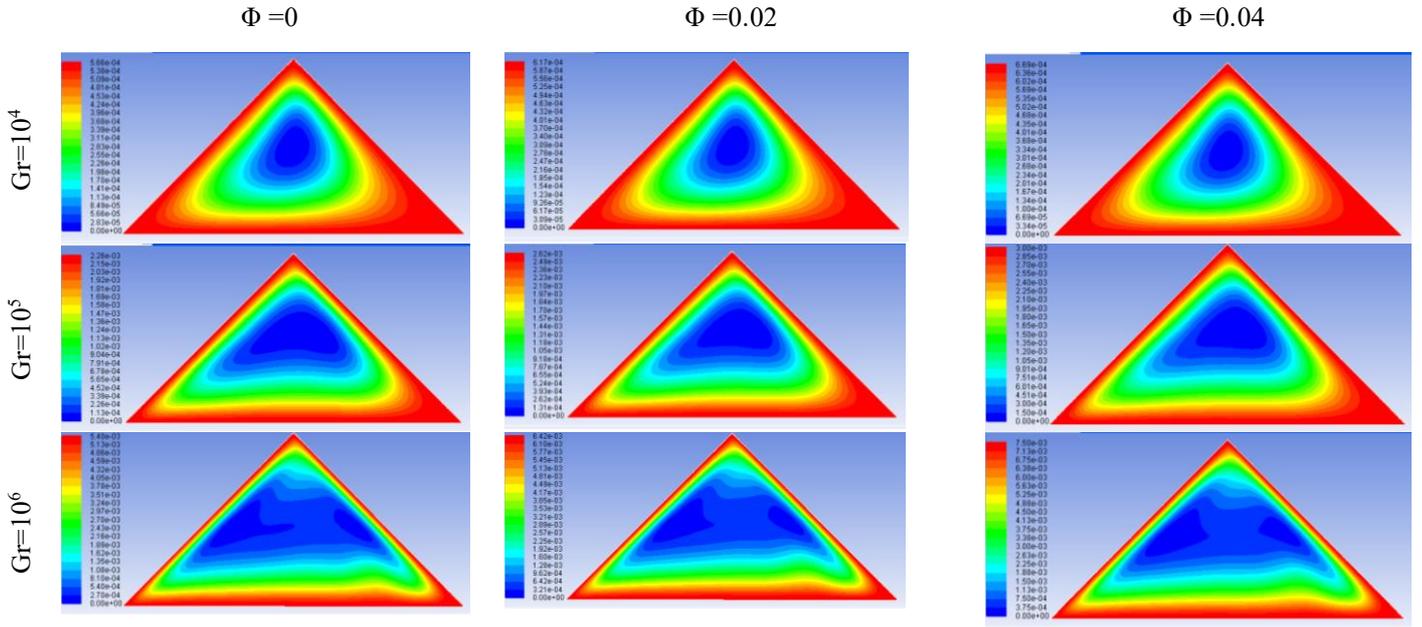


Fig. 2. Streamlines for different Grashof numbers and nanoparticle volume fraction

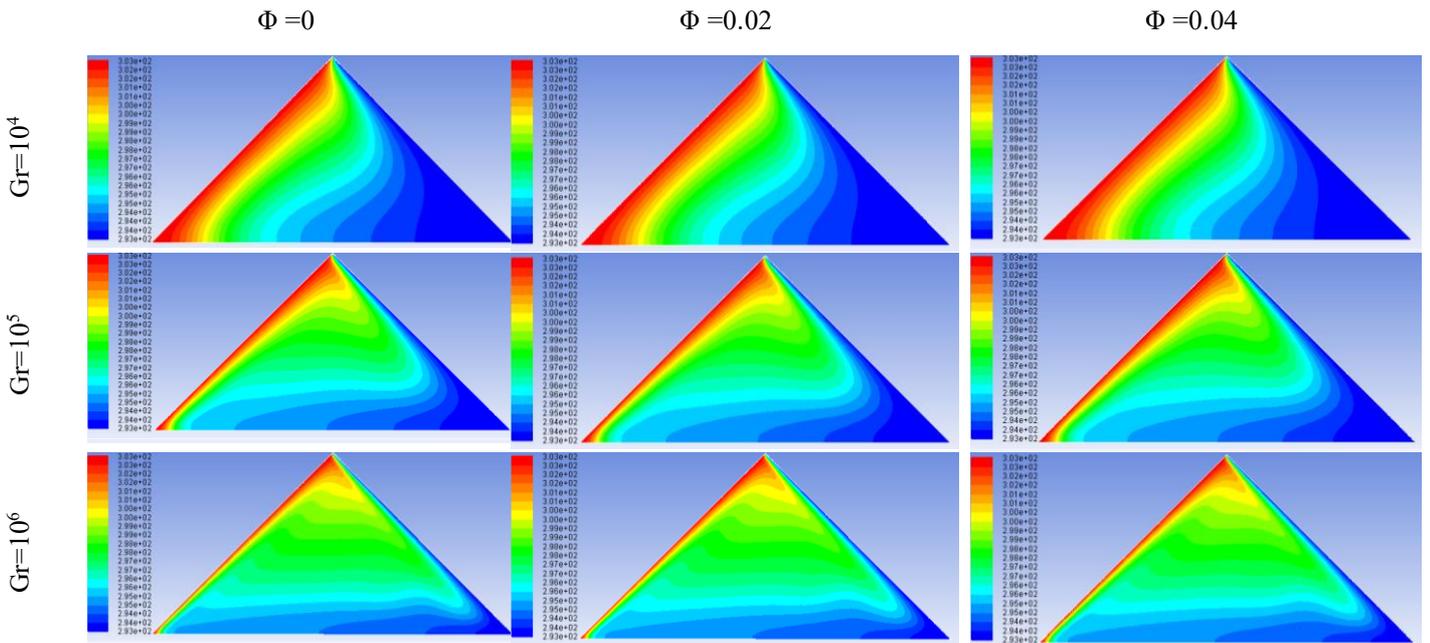


Fig. 3. Isotherms for different Grashof numbers and nanoparticle volume fraction.

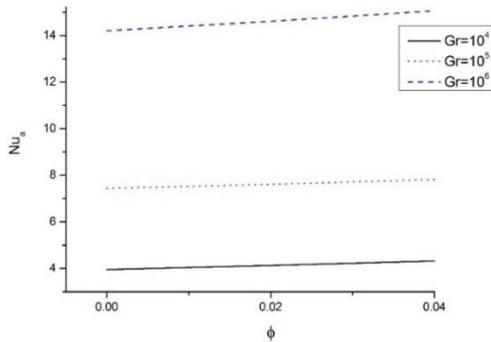


Fig. 4. Variation of mean Nusselt number for Einstein model at different Gr numbers.

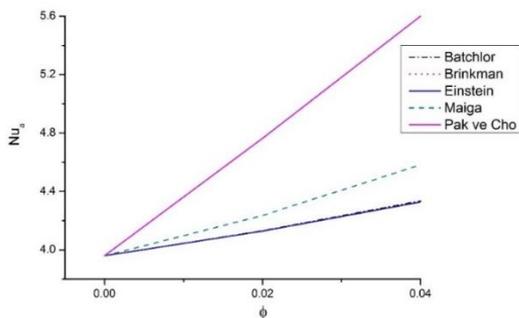


Fig. 5. Variation of mean Nusselt numbers for $Gr=10^4$ at different viscosity models

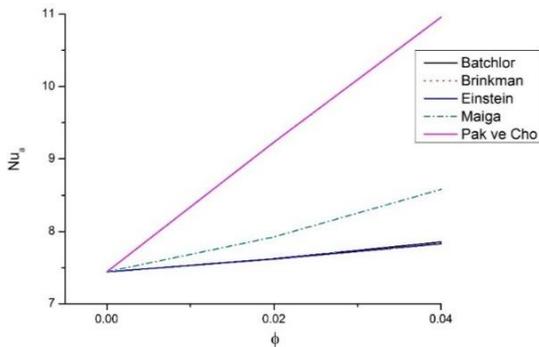


Fig. 6. Variation of mean Nusselt numbers for $Gr=10^5$ at different viscosity models

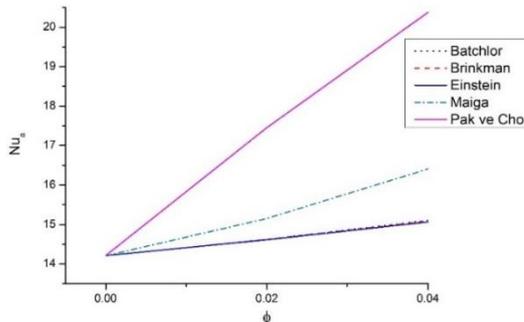
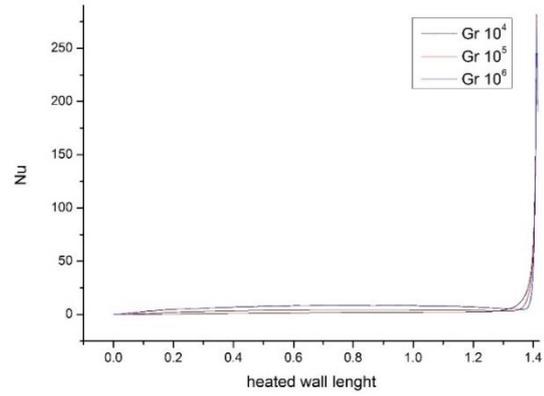
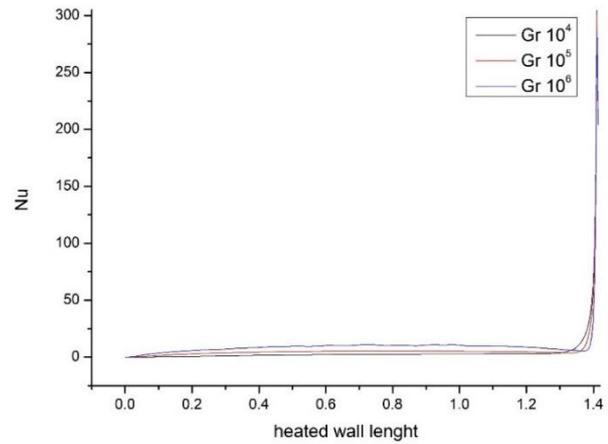


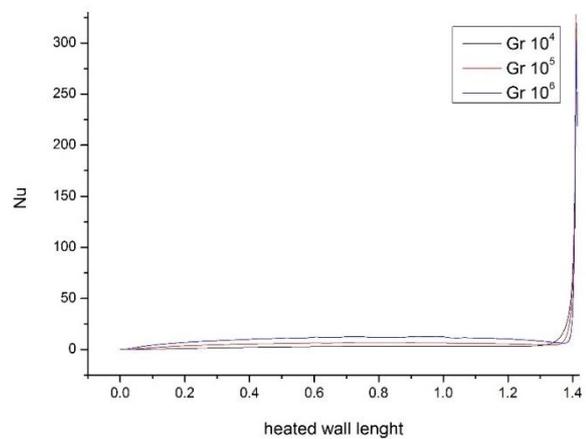
Fig. 7. Variation of mean Nusselt numbers for $Gr=10^6$ at different viscosity models



a)



b)



c) **Fig. 8.** Local Nusselt numbers for a) $\phi=0.0$, b) $\phi=0.02$ and c) $\phi=0.04$ with Pak and Cho viscosity models at different Grashof number

6. References

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